Equations and Graphs of Trigonometric Functions

You can represent phenomena with periodic behaviour or wave characteristics by trigonometric functions or model them approximately with sinusoidal functions. You can identify a trend or pattern, determine an appropriate mathematical model to describe the process, and use it to make predictions (interpolate or extrapolate).

You can use graphs of trigonometric functions to solve trigonometric equations that model periodic phenomena, such as the swing of a pendulum, the motion of a piston in an engine, the motion of a Ferris wheel, variations in blood pressure, the hours of daylight throughout a year, and vibrations that create sounds.

Model Hours of Daylight

Iqaluit is the territorial capital and the largest community of Nunavut. Iqaluit is located at latitude 63° N. The table shows the number of hours of daylight on the 21st day of each month as the day of the year on which it occurs for the capital (based on a 365-day year).

Hours of Daylight by Day of the Year for Iqaluit, Nunavut

Jan 21	Feb 21	Mar 21	Apr 21	May 21	June 21	July 21	Aug 21	Sept 21	Oct 21	Nov 21	Dec 21
21	52	80	111	141	172	202	233	264	294	325	355
6.12	9.36	12.36	15.69	18.88	20.83	18.95	15.69	12.41	9.24	6.05	4.34

max→min
6 months
∴ per = 12 mo.
↓
365

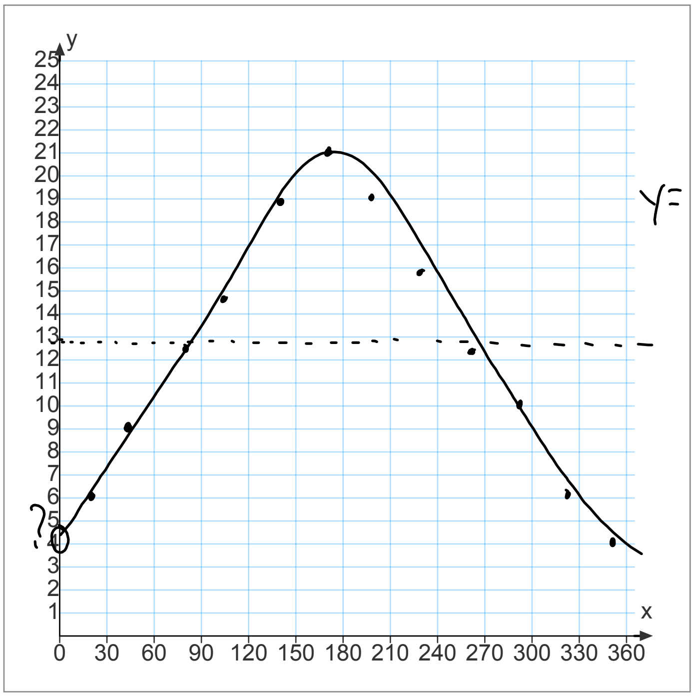
a) Draw a scatter plot for the number of hours of daylight, h, in Iqaluit on the day of the year, t.

b) Which sinusoidal function will best fit the data without requiring a phase shift: $h(t) = \sin t$, $h(t) = -\sin t$, $h(t) = \cos t$, or $h(t) = -\cos t$? Explain.

c) Write the sinusoidal function that models the number of hours of daylight.

d) Graph the function from part c).

e) Estimate the number of hours of daylight on each date.
i) March 15 (day 74) **ii)** July 10 (day 191) **iii)** December 5 (day 339)



b) $y = -\cos(t)$

$y = -8.245\cos\left(\frac{2\pi}{365}t\right) + 12.585$

amp $= \frac{\max - \min}{2}$

$= \frac{20.83 - 4.34}{2}$

$= 8.245$

SA = min + amp OR

Max − amp OR

$\frac{\max + \min}{2}$

$= 12.585$

per = 365

$b = \frac{2\pi}{365}$ OR $\frac{360}{365}$

rad deg

A point on an industrial flywheel experiences a motion described by the function $h(t) = 13 \cos\left(\frac{2\pi}{0.7}t\right) + 15$, where h is the height, in metres, and t is the time, in minutes.

a) What is the maximum height of the point?

b) After how many minutes is the maximum height reached?

c) What is the minimum height of the point?

d) After how many minutes is the minimum height reached?

e) For how long, within one cycle, is the point less than 6 m above the ground?

f) Determine the height of the point if the wheel is allowed to turn for 1 h 12 min.

a) max = amp + SA
$= 13 + 15$
$= 28$ m
$(0, 28)$

b) $28 = 13\cos\left(\frac{2\pi}{0.7}t\right) + 15$
$-15 \qquad\qquad -15$

$\frac{13}{13} = \frac{13\cos\left(\frac{2\pi}{0.7}t\right)}{13}$

$1 = \cos\left(\frac{2\pi}{0.7}t\right)$

per $= \frac{2\pi}{2\pi/0.7}$ $= 0.7$

$\frac{2\pi}{0.7}t = 0$

$t = 0$

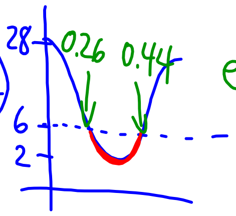
~~$\frac{2\pi}{0.7}t = 2\pi$~~

$t = 0.7$

d) $2 = 13\cos\left(\frac{2\pi}{0.7}t\right) + 15$

Or ½ way from $0 \rightarrow 0.7$
0.35

c) min = SA − amp
$= 15 - 13$
$= 2$
$(0.35, 2)$

e) 

e) $0.44 - 0.26$
$= 0.18$ mins.

CALC
1.VALUE

$h(1\text{hr}\,12\text{min}) = 13\cos\left(\frac{2\pi}{0.7}(72)\right) + 15$
$= 23.1$ m

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#6, 10, 12-16, 19, 21